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ABOUT ONE METHOD OF SOLVING NONHOMOGENEOUS LINEAR DIFFERENTIAL SECOND ORDER EQUATION WITH CONSTANT COEFFICIENTS

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Annotation

The natural laws often are expressed as differential equations, and the calculation of these processes is reduced to solving differential equations. Linear differential equations of the second order with constant coefficients are one of the important objects of investigation of the theory of ordinary differential equations. The paper proposes ways of constructing solutions of second-order linear differential equations with constant coefficients that differ from the classical construction method.

The difference is that they do not use the concept of a complex number in the traditional presentation in the case of a negative discriminant of the characteristic equations of the corresponding linear differential equation with constant coefficients. In the case of a homogeneous differential equation, the essence of the proposed method lies in the construction of linearly independent particular solutions of a linear second order equation by the application of the Bernoulli method to solve a linear first order equation.

Keywords: differential equation, nonhomogeneous differential equation, characteristic equation, partial solution, general solution, method of undetermined coefficients, linear equation, linearly independent solutions.

For a nonhomogeneous linear differential equation of the second order, for which the right-hand side has a special form, methods are used to select the form for recording a particular solution by the form of the right-hand side of the equation, and then the method of undetermined coefficients is applied [1-2].

In papers [3-4] the method of exposition of the topic was presented, which did not attract the concept of a complex number. The proposed work is a continuation of the method for constructing a solution of a nonhomogeneous linear differential equation with constant coefficients in the case of real roots.

Linearly independent solutions of second order homogeneous equation with constant coefficients

A linear nonhomogeneous second order differential equation can be written as

$$y''+py'+qy=f(x), \tag{1}$$

where p and q are constant coefficients.

The general solution of the linear nonhomogeneous equation with constant coefficients is represented as

$$P_n(x)=a_0 \tag{2}$$

where $\bar{y}$ is the general solution of the corresponding homogeneous equation;

$\tilde{y}$ - a particular solution of the nonhomogeneous equation (1).

To find the general solution of equation (1), we can write the general solution of equation

$$y''+py'+qy=0. \tag{3}$$

We solve the related characteristic equation

$$k^2+pk+q=0 \tag{4}$$

and find the roots of the equation

$$k_{1,2}=-\frac{p}{2}\pm\sqrt{D}, \text{ where } D=\frac{p^2}{4}-q.$$

Then the general solution of this second order differential equation with constant coefficients is found as follows:

A) If $D=\frac{p^2}{4}-q>0,$ $k_1\neq k_2,$ then $y_1=e^{k_1x}$, $y_2=e^{k_2x}$ are particular linearly independent solutions, and the general solution is

$$\bar{y}=C_1e^{k_1x}+C_2e^{k_2x}.$$

B) If $D=\frac{p^2}{4}-q=0,$ $k_1=k_2=-\frac{p}{2}$, then $y_1=e^{k_1x}$, $y_2=xe^{k_1x}$ are particular linearly independent solutions, and the general solution is

$$\bar{y}=(C_1+C_2x)e^{k_1x}.$$

C) If $D=\frac{p^2}{4}-q<0,\quad k_{1,2}=-\frac{p}{2}\pm\sqrt{\frac{p^2}{4}-q}=-\frac{p}{2}\pm i\sqrt{q-\frac{p^2}{4}}=\alpha\pm i\beta,$

then $\alpha=-\frac{p}{2},\ \beta=\sqrt{q-\frac{p^2}{4}}$ $y_1=e^{\alpha x}cos\beta x,\ \ y_2=e^{\alpha x}\sin\beta x$ are particular linearly independent solutions, and the general solution has the form

$$\bar{y}=e^{\alpha x}(C_1 cos\beta x+C_2\sin\beta x).$$

In particular, A) If $p\neq 0, q=0,$ then from equation (3) we obtain $y''+py'=0,$ its characteristic equation has the form $k^2+pk=0,$ then $y_1=1,\ \ y_2=e^{-px}$ are particular linearly independent solutions, and the general solution has the form

$$\bar{y}=C_1+C_2e^{-px}.$$

B) If $p=0, q=\omega^2>0,$, then from equation (3) we obtain $y''+\omega^2y=0,$ its characteristic equation looks like $k^2+\omega^2k=0,$ then $y_1=cos\omega x,\ \ y_2=\sin\omega x$ are particular linearly independent solutions, and the general solution has the form $\bar{y}=C_1cos\omega x+C_2\sin\omega x$.

C) If $p=0, q=-\omega^2<0,$ then from equation (3) we obtain $y''-\omega^2y=0,$ its characteristic equation is $k^2-\omega^2k=0,$ then $y_1=e^{-\omega x},\ \ y_2=e^{\omega x}$ particular linearly independent solutions, and the general solution is

$$y=C_1e^{-\omega x}+C_2e^{\omega x}.$$

Below we consider method of constructing a particular solution of a nonhomogeneous equation.

Particular solution of nonhomogeneous equation $y''+py'+qy=P_n(x).$

A nonhomogeneous differential equation of the form

$$y''+py'+qy=P_n(x), \qquad (5)$$

where

$$P_n(x)=a_0x^n+a_1x^{n-1}+a_2x^{n-2}+...+a_{n-2}x^2+a_{n-1}x+a_n \qquad (6)$$

is polynomial of n-th order with given coefficients.

We look for a particular solution $\tilde{y}$ of kind

$$\tilde{y}=x^m\cdot u_n(x), \qquad (7)$$

where m is an unknown number, and

$$u_n(x)=A_0x^n+A_1x^{n-1}+A_2x^{n-2}+\ldots+$$
$$+A_{n-4}x^4+A_{n-3}x^3+A_{n-2}x^2+A_{n-1}x+A_n \qquad (8)$$

is n-th order polynomial with unknown coefficients.

The derivatives of $\tilde{y}$ are

$$\tilde{y}'=mx^{m-1}\cdot u_n(x)+x^m\cdot u_n'(x)$$
$$\tilde{y}''=m(m-1)x^{m-2}\cdot u_n(x)+2\cdot mx^{m-1}\cdot u_n'(x)+x^m\cdot u_n''(x).$$

Substituting in equation (5) we get

$$m(m-1)x^{m-2}\cdot u_n(x)+mx^{m-1}\left[2u_n'(x)+p\cdot u_n(x)\right]+$$
$$+x^m\cdot\left[u_n''(x)+p\cdot u_n'(x)+q\cdot u_n(x)\right]=P_n(x). \qquad (8)$$

We consider various cases:

A) If $q\neq 0$, , i.e. $k_1\neq 0, k_2\neq 0,$ in this case, that equation (8) is true, $m=0,$ we have

$$u_n''(x)+p\cdot u_n'(x)+q\cdot u_n(x)=P_n(x). \qquad (9)$$

Then we find $u_n'(x), u_n''(x)$ and substituting in the equation (9), we obtain

$$q\cdot A_0x^n+(q\cdot A_1+p\cdot n\cdot A_0)x^{n-1}+\left[q\cdot A_2+p\cdot(n-1)\cdot A_1+n(n-1)\cdot A_0\right]x^{n-2}+$$
$$+\left[q\cdot A_3+p\cdot(n-2)\cdot A_2+(n-1)(n-2)\cdot A_1\right]x^{n-3}+\ldots$$
$$+\left(q\cdot A_{n-2}+3p\cdot A_{n-3}+4\cdot 3\cdot A_{n-4}\right)x^2+\left(q\cdot A_{n-1}+2p\cdot A_{n-2}+3\cdot 2\cdot A_{n-3}\right)x+$$
$$\left(q\cdot A_n+p\cdot A_{n-1}+2\cdot 1\cdot A_{n-2}\right)=a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_{n-2}x^2+a_{n-1}x+a_n.$$

Equating the coefficients for the same powers of the variable x, we get the following system

$$\begin{aligned}
x^n: &\quad q\cdot A_0=a_0\\
x^{n-1}: &\quad q\cdot A_1+p\cdot n\cdot A_0=a_1\\
x^{n-2}: &\quad q\cdot A_2+p\cdot(n-1)\cdot A_1+n(n-1)\cdot A_0=a_2\\
x^{n-3}: &\quad q\cdot A_3+p\cdot(n-2)\cdot A_2+(n-1)(n-2)\cdot A_1=a_3\\
&\quad \ldots\\
x^2: &\quad q\cdot A_{n-2}+3p\cdot A_{n-3}+4\cdot 3\cdot A_{n-4}=a_{n-2}\\
x: &\quad q\cdot A_{n-1}+2p\cdot A_{n-2}+3\cdot 2\cdot A_{n-3}=a_{n-1}\\
x^0: &\quad q\cdot A_n+p\cdot A_{n-1}+2\cdot 1\cdot A_{n-2}=a_n
\end{aligned}$$

Solving the system, we find the values of the coefficients. $A_0, A_1, A_2, ..., A_{n-1}, A_n$.
This method is called the method of uncertain coefficients.
So, with $q \neq 0$ a particular solution of equation (5) will be

$$\tilde{y} = A_0 x^n + A_1 x^{n-1} + A_2 x^{n-2} + ... + A_{n-2} x^2 + A_{n-1} x + A_n . \qquad (10)$$

B) If $q = 0$, i.e. $k_1 = 0$ or $k_2 = 0$, then in this case, that equation (8) is satisfied, then from equation (8) $m=1$, we obtain

$$x \cdot u_n''(x) + (2 + xp) \cdot u_n'(x) + p \cdot u_n(x) = P_n(x) . \qquad (11)$$

Furthermore, using the above method of undetermined coefficients, we find $A_0, A_1, A_2, ..., A_{n-1}, A_n$.
Consequently with $q = 0$ a particular solution of equation (5) will be

$$\tilde{y} = A_0 x^{n+1} + A_1 x^n + A_2 x^{n-1} + ... + A_{n-2} x^3 + A_{n-1} x^2 + A_n x . \qquad (12)$$

Sometimes the values of the uundermined coefficients $A_0, A_1, A_2, ..., A_{n-1}, A_n$ are found $\tilde{y}$, by substituting into equation (5) and not using equation (11).
Table below summarizes a general scheme for the formulation of the particular solution of the nonhomogeneous equation (9), i.e., for the equation

$$y'' + py' + qy = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + ... + a_{n-2} x^2 + a_{n-1} x + a_n$$

$q \neq 0$	$\tilde{y} = A_0 x^n + A_1 x^{n-1} + A_2 x^{n-2} + ... + A_{n-2} x^2 + A_{n-1} x + A_n$
$q = 0$	$\tilde{y} = A_0 x^{n+1} + A_1 x^n + A_2 x^{n-1} + ... + A_{n-2} x^3 + A_{n-1} x^2 + A_n x$

The procedure is best illustrated with an example.
Example. Find the general solution of the equation $y'' - 2y' = 12x^2 - 8$.
Solution: The characteristic equation is $k^2 - 2k = 0$, giving $k_1 = 0, k_2 = 2$. Looking at the condition of the problem $q = 0$, the general solution of the homogeneous equation is $\bar{y} = C_1 + C_2 e^{2x}$. The particular solution $\tilde{y}$ will be found as

$$\tilde{y} = x(Ax^2 + Bx + D) = Ax^3 + Bx^2 + Dx.$$

Substituting the derivatives $\tilde{y}' = 3Ax^2 + 2Bx + D$, $\tilde{y}'' = 6Ax + 2B$ in the equation we get

$$6Ax - 2B - 6Ax^2 - 4Bx - 2D = 12x^2 - 8.$$

Using the method of undetermined coefficients yields

$$\begin{array}{llll} x^2: & -6A=12 & A=-2 & \\ x: & 6A-4B=0 & B=-3 & \tilde{y}=-2x^3-3x^2+x \\ x^0: & -2B-2D=-8 & D=1 & \end{array}$$

The general solution of a complete nonhomogeneous differential equation is equal to the sum of its complementary function and any particular solution

$$y=\bar{y}+\tilde{y}=C_1+C_2e^{2x}-2x^3-3Bx^2+x.$$

In particular, if $P_n(x)=a_0$, then the particular solution $\tilde{y}$ for the equation $y''+py'+qy=a_0$ will be

$$\tilde{y}=\frac{a_0}{q}, \text{ if } q\neq 0, \qquad \tilde{y}=\frac{a_0x}{p}, \text{ if } q=0.$$

Particular solution of non-homogeneous equation $y''+py'+qy=a_0\cdot e^{\alpha x}$

Special case of nonhomogeneous differential equation can be written in the form

$$y''+py'+qy=a_0\cdot e^{\alpha x}, \tag{14}$$

where a_0,α - given numbers.

The particular solution $\tilde{y}$ is of the form

$$\tilde{y}=A\cdot x^m\cdot e^{\alpha x}, \tag{15}$$

где m, A - unknown constants.

With the derivatives $\tilde{y}$

$$y'=A\left(mx^{m-1}+\alpha x^m\right)\cdot e^{\alpha x}$$

$$y''=A\left[m(m-1)x^{m-2}+2\alpha mx^{m-1}+\alpha^2x^m\right]\cdot e^{\alpha x}$$

we obtain

$$A\left[m(m-1)x^{m-2}+m(2\alpha+p)x^{m-1}+\left(\alpha^2+p\alpha+q\right)x^m\right]=a_0. \tag{16}$$

We consider various cases:

A) If α is not the root of the characteristic equation (4), i.e. $k_1\neq\alpha, k_2\neq\alpha, \alpha^2+p\alpha+q\neq 0$, then in this case, equation for $m=0$, is fulfilled and from equation (14) we get

$$A\left(a^2 + pa + q\right) = a_0 \quad \text{or} \qquad A = \frac{a_0}{a^2 + pa + q}.$$

The particular solution of the equation (14) is therefore

$$\tilde{y} = \frac{a_0}{a^2 + pa + q} e^{ax}. \tag{17}$$

B) If a is the simple root of the characteristic equation (4), i.e. $k_1 = a, \;\; k_2 \neq a, \; a^2 + pa + q = 0, \; 2a + p \neq 0,$ then in this case, equation (14) $m=1$, is fulfilled and from equation (14), we obtain $A\left(2a + p\right) = a_0$ or $A = \frac{a_0}{2a + p}$.

In this case, the particular solution of equation (14) will be

$$\tilde{y} = \frac{a_0}{2a + p} x e^{ax}. \tag{18}$$

C) If a is a simple root of the characteristic equation (4), i.e. $k_1 = a, \;\; k_2 = a, \; a^2 + pa + q = 0, \; 2a + p = 0,$ then in this case, equation (14) $m=2$, is fulfilled and from equation (14) we get

$$2A = a_0 \quad \text{or} \qquad A = \frac{a_0}{2}.$$

Then the particular solution of equation (14) will be

$$\tilde{y} = \frac{a_0}{2} x^2 e^{ax}. \tag{19}$$

Table below summarizes a general scheme for the formulation of the particular solution of the nonhomogeneous equation (14), i.e., for the equation $y'' + py' + qy = a_0 \cdot e^{ax}$

$k_1 \neq a, \;\; k_2 \neq a$	$\tilde{y} = \frac{a_0}{a^2 + pa + q} e^{ax}$
$k_1 = a, \;\; k_2 \neq a$	$\tilde{y} = \frac{a_0}{2a + p} x e^{ax}$
$k_1 = k_2 = a$	$\tilde{y} = \frac{a_0}{2} x^2 e^{ax}$

The procedure is best illustrated with the following examples.

1. Find the general solution of the equation $y'' + 7y' - 8y = 4e^{2x}$.

Solution: The characteristic equation has the form $k^2+7k-8=0$, its solution $k_1=1, k_2=-8$. According to the condition of the problem $a_0=4, a=2, k_1 \neq a, k_2 \neq a$, that is, a is not the root of the characteristic equation (4). Then the general solution of the homogeneous equation $y''+7y'-8y=0$ is $\bar{y}=C_1e^x+C_2e^{-8x}$, and a particular solution is

$$\tilde{y}=\frac{a_0}{a^2+pa+q}e^{ax}=0{,}4e^{2x}.$$

The general solution to the nonhomogeneous differential equation is therefore

$$y=\bar{y}+\tilde{y}=C_1e^x+C_2e^{-8x}+0{,}4e^{2x}.$$

2. Find the general solution of the equation $y''-3y'-4y=4e^{-x}$.

Solution: The characteristic equation has the form $k^2-3k-4=0$, its solution $k_1=-1, k_2=4$, by the condition of the problem $a_0=4, a=-1, k_1=a, k_2 \neq a$, that is, a is a simple root of the characteristic equation (4). Then the general solution of the homogeneous equation $y''-3y'-4y=0$ is $\bar{y}=C_1e^{-x}+C_2e^{4x}$, and the particular solution is

$$\tilde{y}=\frac{a_0}{2a+p}xe^{ax}=-0{,}8xe^{-x}.$$

The general solution to the nonhomogeneous differential equation is therefore

$$y=\bar{y}+\tilde{y}=C_1e^{-x}+C_2e^{4x}-0{,}8xe^{-x}=(C_1-0{,}8x)e^{-x}+C_2e^{4x}.$$

3. Find the general solution of the equation $y''-4y'+4y=6e^{2x}$.

Solution: The characteristic equation has the form $k^2-4k+4=0$, its solution $k_1=k_2=2$, by the condition of the problem $a_0=6, a=2, k_1=k_2=a$, that is, a is a multiple root of the characteristic equation (4). Then the general solution of the homogeneous equation $y''-4y'+4y=0$ is $\bar{y}=(C_1x+C_2)e^{2x}$, and the particular solution is

$$\tilde{y}=\frac{a_0}{2}x^2e^{ax}=3x^2e^{2x}.$$

The general solution to the nonhomogeneous differential equation is therefore

$$y=\bar{y}+\tilde{y}=(C_1x+C_2)e^{2x}+3x^2e^{2x}=(3x^2+C_1x+C_2)e^{2x}.$$

Particular solution of nonhomogeneous equation $y''+py'+qy=P_n(x)\cdot e^{ax}$.

Special case of the linear nonhomogeneous equation of the form

$$y'' + py' + qy = P_n(x) \cdot e^{ax}, \tag{20}$$

where $P_n(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_{n-2} x^2 + a_{n-1} x + a_n$ is a nth order polynomial with given coefficients, a is constant given number.

We look for a particular solution $\tilde{y}$ in the form $\tilde{y} = x^m \cdot u_n(x) \cdot e^{ax}$, where $u_n(x) = A_0 x^n + A_1 x^{n-1} + A_2 x^{n-2} + \ldots + A_{n-2} x^2 + A_{n-1} x + A_n$ is a nth order polynomial with undetermined coefficients, m is an constant number.

With the derivatives $\tilde{y}$

$$\tilde{y}' = m x^{m-1} \cdot u_n(x) \cdot e^{ax} + x^m \left[u_n'(x) + a u_n(x)\right] e^{ax},$$

$$\tilde{y}'' = m(m-1) x^{m-2} \cdot u_n(x) \cdot e^{ax} + 2m x^{m-1} \left[u_n'(x) + a \cdot u_n(x)\right] e^{ax} +$$
$$x^m \cdot \left[u_n''(x) + 2a \cdot u_n'(x) + a^2 \cdot u_n(x)\right] e^{ax}$$

we obtain

$$\begin{aligned} & m(m-1) x^{m-2} \cdot u_n(x) + m x^{m-1} \left[2u_n'(x) + (2a + p) u_n(x)\right] + \\ & x^m \cdot \left[u_n''(x) + (2a + p) u_n'(x) + \left(a^2 + pa + q\right) \cdot u_n(x)\right] = P_n(x) \end{aligned} . \tag{21}$$

We consider various cases:

A) If a is not the root of the characteristic equation (4), i.e. $k_1 \neq a, k_2 \neq a, a^2 + pa + q \neq 0,$ then in this case $m = 0,$, that equation (21) is fulfilled and from equation (21) we get

$$u_n''(x) + (2a + p) \cdot u_n'(x) + \left(a^2 + pa + q\right) \cdot u_n(x) = P_n(x) . \tag{22}$$

Therefore, a particular solution of equation (21) will be

$$\tilde{y} = u_n \cdot e^{ax} = \left(A_0 x^n + A_1 x^{n-1} + A_2 x^{n-2} + \ldots + A_{n-2} x^2 + A_{n-1} x + A_n\right) e^{ax} . \tag{23}$$

B) If a is the simple root of the characteristic equation (4), i.e. $k_1 = a,\ \ k_2 \neq a,\ a^2 + pa + q = 0,\ 2a + p \neq 0,$ then in this case $m = 1,$ equation (21) is fulfilled and from equation (21) we get

$$2u_n'(x) + (2a + p) \cdot u_n(x) + x\left[u_n''(x) + (2a + p) \cdot u_n'(x)\right] = P_n(x) . \tag{24}$$

Therefore, a particular solution of equation (21) will be

$$\begin{aligned} & \tilde{y} = x \cdot u_n \cdot e^{ax} = \\ & = \left(A_0 x^{n+1} + A_1 x^n + A_2 x^{n-1} + \ldots + A_{n-2} x^3 + A_{n-1} x^2 + A_n x\right) e^{ax} \end{aligned} . \tag{25}$$

C) If a is a simple root of the characteristic equation (4), i.e. $k_1 = a$, $k_2 = a$, $a^2 + pa + q = 0$, $2a + p = 0$, then in this case, equation (21) $m=2$, is fulfilled and from equation (21) we get

$$u_n(x) + 4x\cdot u_n'(x) + x^2\cdot u_n''(x) = P_n(x). \qquad (26)$$

Therefore, a particular solution of equation (14) will have the form

$$\tilde{y} = x^2\cdot u_n\cdot e^{ax} = \\ =\left(A_0x^{n+2} + A_1x^{n+1} + A_2x^n + ... + A_{n-2}x^4 + A_{n-1}x^3 + A_nx^2\right)e^{ax}. \qquad (27)$$

Furthermore, using the above method of undetermined coefficients, we can find the values of the undermined coefficients $A_0, A_1, A_2, ..., A_{n-1}, A_n$.

Table below summarizes a general scheme for the formulation of the particular solution of the nonhomogeneous equation (20), i.e., for the equation $y'' + py' + qy = P_n(x)\cdot e^{ax}$, where $P_n(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-2}x^2 + a_{n-1}x + a_n$.

$k_1 \neq a$, $k_2 \neq a$	$\tilde{y} = u_n\cdot e^{ax} = \left(A_0x^n + A_1x^{n-1} + A_2x^{n-2} + ... + A_{n-2}x^2 + A_{n-1}x + A_n\right)e^{ax}$
$k_1 = a$, $k_2 \neq a$	$\tilde{y} = x\cdot u_n\cdot e^{ax} = \left(A_0x^{n+1} + A_1x^n + A_2x^{n-1} + ... + A_{n-2}x^3 + A_{n-1}x^2 + A_nx\right)e^{ax}$
$k_1 = k_2 = a$	$\tilde{y} = x^2\cdot u_n\cdot e^{ax} = \left(A_0x^{n+2} + A_1x^{n+1} + A_2x^n + ... + A_{n-2}x^4 + A_{n-1}x^3 + A_nx^2\right)e^{ax}$

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ТҰРАҚТЫ КОЭФФИЦИЕНТТІ СЫЗЫҚТЫҚ ДИФФЕРЕНЦИАЛДЫҚ ТЕҢДЕУЛЕРДІҢ ЖӘНЕ ОЛАРДЫҢ ЖҮЙЕЛЕРІНІҢ ШЕШІМДЕРІН ҚҰРУ

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Түйін

Қандай да бір процеске бағынатын табиғат заңы көп жағдайларда дифференциалдық теңдеулер түрінде беріледі, ал осы процестердің қозғалыс ағымдарының шешімін дифференциалдық теңдеулердің шешімі арқылы табылады. Екінші ретті тұрақты коэффициентті сызықты дифференциалдық теңдеулер қарапайым дифференциалдық теңдеулер теориясының зерттейтін ең маңызды нысандарының бірі болып табылады. Жұмыста екінші ретті тұрақты коэффициентті дифференциалдық теңдеулердің және олардың жүйелерінің шешімдерін дәстүрлік әдістемеден ерекшеленетін жолмен құру ұсынылады. Олар дифференциалдық теңдеулер немесе теңдеулер жүйесіндегі сипаттамалық теңдеудің дискримианты теріс болған жағдайында қолданылатын дәстүрлік есептелулердегі комплекс сан ұғымының

қолданбауымен ерекшелінеді. Ұсынылған әдістің мағынасы бойынша, біртекті теңдеулер жағдайында, екінші ретті сызықтық теңдеулердің сызықтық тәуелсіз дербес шешімдерін құруда бірінші ретті сызықтық теңдеулерді шешуде кездесетін Бернулли әдісінің көмегімен енгізілуінде, сондай-ақ алынған алгоритм біртекті теңдеулер жүйесінің шешімін құруда да қолданылады.

Кілттік сөздер: дифференциалдық теңдеу,бір текті емес дифференциалдық теңдеу, сипаттамалық теңдеулер, дербес шешімі, жалпы шешім, белгісіз коэффициеттер әдісі, сызықты теңдеу, сызықты тәуелсіз шешімдер.

ОБ ОДНОМ МЕТОДЕ РЕШЕНИЯ НЕОДНОРОДНОГО ЛИНЕЙНОГО ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ ВТОРОГО ПОРЯДКА С ПОСТОЯННЫМИ КОЭФФИЦИЕНТАМИ

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Резюме

Законы природы, управляющие теми или иными процессами, часто выражаются в виде дифференциальных уравнений, а расчет течения этих процессов сводится к решению дифференциальных уравнений. Линейные дифференциальные уравнения второго порядка с постоянными коэффициентами являются одним из важных объектов исследования теории обыкновенных дифференциальных уравнений. В работе предлагаются пути построения решений линейных дифференциальных уравнений второго порядка с постоянными коэффициентами, отличающиеся от классической методики построения. Отличие состоит в том, что в них не используется понятие комплексного числа, применяемого в традиционном изложении в случае отрицательного дискриминанта характеристических уравнений соответствующего линейного дифференциального уравнения с постоянными коэффициентами. В случае однородного дифференциального уравнения суть предлагаемого метода заключается в использовании при построении линейно независимых частных решений линейного уравнения второго порядка метода Бернулли, применяемого для решения линейного уравнения первого порядка.

Ключевые слова: дифференциальное уравнение, неоднородное дифференциальное уравнение, характеристическое уравнение, частное решение, общее решение, метод неопределенных коэффициентов, линейное уравнение, линейно независимые решения.